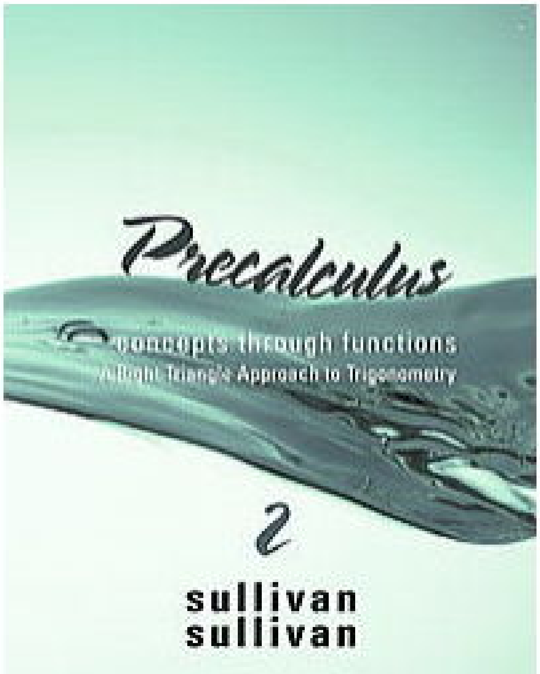
Pre-Calculus Summer Math Packet (2019)



Book is available on MBS. In the past, Parents have also found the book available elsewhere on-line. I would look on-line first before purchasing.

- This work is due the 2nd day of school. The first day of school I will answer questions regarding the work.
- Please do not wait until the night before, day before, or weekend before school starts to do this!
- ALL WORK MUST BE SHOWN!
- The packet contains a brief review and example problems for each skill.
- If students need more explanation and practice, they should read the corresponding sections in the Appendix A chapter of the text. (Answers to even and odd numbered problems are in the back of the book.) Appendix topics: A.1 Algebra Essentials; A.3 Polynomials; A.4 Factoring Polynomials; A.6 Rational Expressions; A.8 Solving Equations
- Students will take a quiz covering this material in the first week of school.
- Should you lose this packet, it can be found on the school web page.

Have a good summer. I look forward to seeing you in August.

Ms. Zareva

Laws of Exponents

$$a^m a^n = a^{m+n} \qquad (a^m)^n = a^{mn} \qquad (ab)^n = a^n b^n$$

$$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}} \quad \text{if } a \neq 0 \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \text{if } b \neq 0$$

If $a \neq 0$ and if n is a positive integer, then we define

$$a^{-n} = \frac{1}{a^n} \quad \text{if } a \neq 0$$

Using the Laws of Exponents

Write each expression so that all exponents are positive.

(a) $\dfrac{x^5 y^{-2}}{x^3 y} \quad x \neq 0, \quad y \neq 0$ (b) $\left(\dfrac{x^{-3}}{3y^{-1}}\right)^{-2} \quad x \neq 0, \quad y \neq 0$

(a) $\dfrac{x^5 y^{-2}}{x^3 y} = \dfrac{x^5}{x^3} \cdot \dfrac{y^{-2}}{y} = x^{5-3} \cdot y^{-2-1} = x^2 y^{-3} = x^2 \cdot \dfrac{1}{y^3} = \dfrac{x^2}{y^3}$

(b) $\left(\dfrac{x^{-3}}{3y^{-1}}\right)^{-2} = \dfrac{(x^{-3})^{-2}}{(3y^{-1})^{-2}} = \dfrac{x^6}{3^{-2}(y^{-1})^{-2}} = \dfrac{x^6}{\frac{1}{9}y^2} = \dfrac{9x^6}{y^2}$

Know Formulas for Special Products

Difference of Two Squares	$x^2 - a^2 = (x - a)(x + a)$
Perfect Squares	$x^2 + 2ax + a^2 = (x + a)^2$
	$x^2 - 2ax + a^2 = (x - a)^2$
Sum of Two Cubes	$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$
Difference of Two Cubes	$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

To factor a second-degree polynomial $x^2 + Bx + C$, find integers whose product is C and whose sum is B. That is, if there are numbers a, b, where $ab = C$ and $a + b = B$, then

$$x^2 + Bx + C = (x + a)(x + b)$$

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

12 is the product of 3 and 4

7 is the sum of 3 and 4

Steps for Factoring $Ax^2 + Bx + C$, when $A \neq 1$ and A, B, and C Have No Common Factors

Step 1: Find the value of AC.
Step 2: Find integers whose product is AC that add up to B. That is, find a and b so that $ab = AC$ and $a + b = B$.
Step 3: Write $Ax^2 + Bx + C = Ax^2 + ax + bx + C$.
Step 4: Factor this last expression by grouping.

Factor completely: $2x^2 + 5x + 3$

Step 1: The value of AC is $2 \cdot 3 = 6$.
Step 2: Determine the integers whose product is $AC = 6$ and compute their sums.

Step 3: The integers whose product is 6 that add up to $B = 5$ are 2 and 3.

$$2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$$

Step 4: Factor by grouping.

$$\begin{aligned} 2x^2 + 2x + 3x + 3 &= (2x^2 + 2x) + (3x + 3) \\ &= 2x(x + 1) + 3(x + 1) \\ &= (x + 1)(2x + 3) \end{aligned}$$

As a result,

$$2x^2 + 5x + 3 = (x + 1)(2x + 3)$$

SUMMARY

Type of Polynomial	Method	Example
Any polynomial	Look for common monomial factors. (Always do this first!)	$6x^2 + 9x = 3x(2x + 3)$
Binomials of degree 2 or higher	Check for a special product:	
	Difference of two squares, $x^2 - a^2$	$x^2 - 16 = (x - 4)(x + 4)$
	Difference of two cubes, $x^3 - a^3$	$x^3 - 64 = (x - 4)(x^2 + 4x + 16)$
	Sum of two cubes, $x^3 + a^3$	$x^3 + 27 = (x + 3)(x^2 - 3x + 9)$
Trinomials of degree 2	Check for a perfect square, $(x \pm a)^2$	$x^2 + 8x + 16 = (x + 4)^2$
		$x^2 - 10x + 25 = (x - 5)^2$
	Factoring $x^2 + Bx + C$ (p. A36)	$x^2 - x - 2 = (x - 2)(x + 1)$
	Factoring $Ax^2 + Bx + C$ (p. A38)	$6x^2 + x - 1 = (2x + 1)(3x - 1)$
Four or more terms	Grouping	$2x^3 - 3x^2 + 4x - 6 = (2x - 3)(x^2 + 2)$

A rational expression is reduced to lowest terms by factoring completely the numerator and the denominator and canceling any common factors by using the Cancellation Property:

$$\frac{a\cancel{c}}{b\cancel{c}} = \frac{a}{b} \quad \text{if } b \neq 0, c \neq 0 \qquad \textbf{(1)}$$

Reducing Rational Expressions to Lowest Terms

(a) $\dfrac{x^3 - 8}{x^3 - 2x^2} = \dfrac{\cancel{(x - 2)}(x^2 + 2x + 4)}{x^2\cancel{(x - 2)}} = \dfrac{x^2 + 2x + 4}{x^2} \qquad x \neq 0, 2$

(b) $\dfrac{8 - 2x}{x^2 - x - 12} = \dfrac{2(4 - x)}{(x - 4)(x + 3)} = \dfrac{2(-1)\cancel{(x - 4)}}{\cancel{(x - 4)}(x + 3)} = \dfrac{-2}{x + 3} \qquad x \neq -3, 4$

The LCM Method for Adding or Subtracting Rational Expressions

The Least Common Multiple (LCM) Method requires four steps:

Step 1: Factor completely the polynomial in the denominator of each rational expression.

Step 2: The LCM of the denominator is the product of each of these factors raised to a power equal to the greatest number of times that the factor occurs in the polynomials.

Step 3: Write each rational expression using the LCM as the common denominator.

Step 4: Add or subtract the rational expressions using equation (4).

Using the Least Common Multiple to Add Rational Expressions

Perform the indicated operation and simplify the result. Leave your answer in factored form.

$$\frac{x}{x^2 + 3x + 2} + \frac{2x - 3}{x^2 - 1} \qquad x \neq -2, -1, 1$$

Step 1: Factor completely the polynomials in the denominators.

$$x^2 + 3x + 2 = (x + 2)(x + 1)$$
$$x^2 - 1 = (x - 1)(x + 1)$$

Step 2: The LCM is $(x + 2)(x + 1)(x - 1)$. Do you see why?

Step 3: Write each rational expression using the LCM as the denominator.

$$\frac{x}{x^2 + 3x + 2} = \frac{x}{(x + 2)(x + 1)} = \frac{x}{(x + 2)(x + 1)} \cdot \frac{x - 1}{x - 1} = \frac{x(x - 1)}{(x + 2)(x + 1)(x - 1)}$$

Multiply numerator and denominator by $x - 1$ to get the LCM in the denominator.

$$\frac{2x - 3}{x^2 - 1} = \frac{2x - 3}{(x - 1)(x + 1)} = \frac{2x - 3}{(x - 1)(x + 1)} \cdot \frac{x + 2}{x + 2} = \frac{(2x - 3)(x + 2)}{(x - 1)(x + 1)(x + 2)}$$

Multiply numerator and denominator by $x + 2$ to get the LCM in the denominator.

Step 4: Now we can add by using equation (4).

$$\frac{x}{x^2 + 3x + 2} + \frac{2x - 3}{x^2 - 1} = \frac{x(x - 1)}{(x + 2)(x + 1)(x - 1)} + \frac{(2x - 3)(x + 2)}{(x + 2)(x + 1)(x - 1)}$$
$$= \frac{(x^2 - x) + (2x^2 + x - 6)}{(x + 2)(x + 1)(x - 1)}$$
$$= \frac{3x^2 - 6}{(x + 2)(x + 1)(x - 1)} = \frac{3(x^2 - 2)}{(x + 2)(x + 1)(x - 1)}$$

Simplifying a Complex Rational Expression

Simplify: $\dfrac{\dfrac{1}{2}+\dfrac{3}{x}}{\dfrac{x+3}{4}} \qquad x \neq -3, 0$

Method 1: First, perform the indicated operation in the numerator, and then divide.

$$\frac{\frac{1}{2}+\frac{3}{x}}{\frac{x+3}{4}} = \frac{\frac{1\cdot x+2\cdot 3}{2\cdot x}}{\frac{x+3}{4}} = \frac{\frac{x+6}{2x}}{\frac{x+3}{4}} = \frac{x+6}{2x}\cdot\frac{4}{x+3}$$

Rule for adding quotients   Rule for dividing quotients

$$= \frac{(x+6)\cdot 4}{2\cdot x\cdot(x+3)} = \frac{\cancel{2}\cdot 2\cdot(x+6)}{\cancel{2}\cdot x\cdot(x+3)} = \frac{2(x+6)}{x(x+3)}$$

Rule for multiplying quotients

Simplifying a Complex Rational Expression

Simplify: $\dfrac{\dfrac{x^2}{x-4}+2}{\dfrac{2x-2}{x}-1} \quad x \neq 0, 2, 4$

We will use Method 1.

$$\frac{\frac{x^2}{x-4}+2}{\frac{2x-2}{x}-1} = \frac{\frac{x^2}{x-4}+\frac{2(x-4)}{x-4}}{\frac{2x-2}{x}-\frac{x}{x}} = \frac{\frac{x^2+2x-8}{x-4}}{\frac{2x-2-x}{x}}$$

$$= \frac{\frac{(x+4)(x-2)}{x-4}}{\frac{x-2}{x}} = \frac{(x+4)\cancel{(x-2)}}{x-4}\cdot\frac{x}{\cancel{x-2}} = \frac{(x+4)\cdot x}{x-4}$$

Solve Rational Equations

To solve a rational equation, multiply both sides of the equation by the least common multiple of the denominators of the rational expressions that make up the rational equation.

Solving a Rational Equation

Solve the equation: $\frac{3}{x-2} = \frac{1}{x-1} + \frac{7}{(x-1)(x-2)}$

First, we note that the domain of the variable is $\{x | x \neq 1, x \neq 2\}$. We clear the equation of rational expressions by multiplying both sides by the least common multiple of the denominators of the three rational expressions, $(x-1)(x-2)$.

$$\frac{3}{x-2} = \frac{1}{x-1} + \frac{7}{(x-1)(x-2)}$$

$$(x-1)\cancel{(x-2)}\frac{3}{\cancel{x-2}} = (x-1)(x-2)\left[\frac{1}{x-1} + \frac{7}{(x-1)(x-2)}\right]$$ Multiply both sides by $(x-1)(x-2)$. Cancel on the left.

$$3x - 3 = \cancel{(x-1)}(x-2)\frac{1}{\cancel{x-1}} + \cancel{(x-1)}\cancel{(x-2)}\frac{7}{\cancel{(x-1)}\cancel{(x-2)}}$$ Use the Distributive Property on each side; cancel on the right.

$$3x - 3 = (x-2) + 7$$ Rewrite the equation.

$$3x - 3 = x + 5$$ Combine like terms.

$$2x = 8$$ Add 3 to each side; subtract x from each side.

$$x = 4$$ Divide by 2.

A Rational Equation with No Solution

Solve the equation: $\frac{3x}{x-1} + 2 = \frac{3}{x-1}$

First, we note that the domain of the variable is $\{x | x \neq 1\}$. Since the two rational expressions in the equation have the same denominator, $x-1$, we can simplify by multiplying both sides by $x-1$. The resulting equation is equivalent to the original equation, since we are multiplying by $x-1$, which is not 0 (remember, $x \neq 1$).

$$\frac{3x}{x-1} + 2 = \frac{3}{x-1}$$

$$\left(\frac{3x}{x-1} + 2\right)\cdot(x-1) = \frac{3}{\cancel{x-1}}\cdot\cancel{(x-1)}$$ Multiply both sides by x − 1; cancel on the right.

$$\frac{3x}{\cancel{x-1}}\cdot\cancel{(x-1)} + 2\cdot(x-1) = 3$$ Use the Distributive Property on the left side; cancel on the left.

$$3x + 2x - 2 = 3$$ Simplify.

$$5x - 2 = 3$$ Combine like terms.

$$5x = 5$$ Add 2 to each side.

$$x = 1$$ Divide both sides by 5.

The solution appears to be 1. But recall that $x = 1$ is not in the domain of the variable, so $x = 1$ is an extraneous solution. The equation has no solution. The solution set is $\varnothing$.

Solve Equations By Factoring

Many equations can be solved by factoring and then using the Zero-Product Property.

Solving Equations by Factoring

Solve the equations:

(a) $x^2 = 4x$ (b) $x^3 - x^2 - 4x + 4 = 0$

(a) We begin by collecting all terms on one side. This results in 0 on one side and an expression to be factored on the other.

$$x^2 = 4x$$
$$x^2 - 4x = 0$$
$$x(x - 4) = 0 \quad \text{Factor.}$$
$$x = 0 \quad \text{or} \quad x - 4 = 0 \quad \text{Apply the Zero-Product Property.}$$
$$x = 4$$

The solution set is $\{0, 4\}$.

(b) We group the terms of $x^3 - x^2 - 4x + 4 = 0$ as follows:

$$(x^3 - x^2) - (4x - 4) = 0$$

Factor out x^2 from the first grouping and 4 from the second.

$$x^2(x - 1) - 4(x - 1) = 0$$

This reveals the common factor $(x - 1)$, so we have

$$(x^2 - 4)(x - 1) = 0$$
$$(x - 2)(x + 2)(x - 1) = 0 \quad \text{Factor again.}$$
$$x - 2 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{or} \quad x - 1 = 0 \quad \text{Set each factor equal to 0.}$$
$$x = 2 \qquad x = -2 \qquad x = 1 \quad \text{Solve.}$$

The solution set is $\{-2, 1, 2\}$.

Solving a Radical Equation

Find the real solutions of the equation: $\sqrt{x - 1} = x - 7$

We square both sides since the index of a square root is 2.

$$\sqrt{x - 1} = x - 7$$
$$\left(\sqrt{x - 1}\right)^2 = (x - 7)^2 \quad \text{Square both sides.}$$
$$x - 1 = x^2 - 14x + 49 \quad \text{Remove parentheses.}$$
$$x^2 - 15x + 50 = 0 \quad \text{Put in standard form.}$$
$$(x - 10)(x - 5) = 0 \quad \text{Factor.}$$
$$x = 10 \quad \text{or} \quad x = 5 \quad \text{Apply the Zero-Product Property and solve.}$$

Check: $x = 10$: $\sqrt{x - 1} = \sqrt{10 - 1} = \sqrt{9} = 3$ and $x - 7 = 10 - 7 = 3$

$x = 5$: $\sqrt{x - 1} = \sqrt{5 - 1} = \sqrt{4} = 2$ and $x - 7 = 5 - 7 = -2$

The apparent algebraic solution $x = 5$ is extraneous; the only solution of the equation is $x = 10$. The solution set is $\{10\}$.

Simplify each expression. Write your answer without negative exponents.

1. $(-4x^2)^{-1}$

2. $\frac{4x^{-2}(yz)^{-1}}{2^3x^4y}$

3. $\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

Factor completely each expression.

4. $4x^2 - 25$

5. $2x^2 + 5x - 12$

6. $x^3 - 3x^2 - 4x + 12$

7. $x^4 + 27x$

8. $6x^2 + 8x + 2$

9. $x^4 - 1$

10. $x^8 - x^5$

11. $4x^2 - 16x + 15$

12. $(x + 2)^2 - 5(x + 2)$

13. $(5x + 1)^3 - 1$

14. $3(x^2 + 10x + 25) - 4(x + 5)$

15. $x^3 - 3x^2 - x + 3$

Perform the indicated operations. Simplify your result.

16. $\frac{x^2+11x+30}{x^2+15x+56} \cdot \frac{x^2+4x-32}{3x^2+18x}$

17. $\frac{2x^2-x-6}{x^3+2x^2} \cdot \frac{6x+12}{3x^2-12}$

18. $\frac{x^3-8}{3x^2-10x+8} \cdot \frac{6x^2-8x}{2x^3+4x^2+8x}$

19. $\frac{x^2}{x^2-1} \div \frac{3x^2-x}{3x^2-4x+1}$

20. $\frac{2x+1}{16x^2} \div \frac{2x^2+5x+2}{4x^3+4x}$

21. $\frac{x^3+4x}{2x-1} \div \frac{x^4-16}{x^2-4x-12}$

22. $\frac{x+2}{x^2-1} \cdot \frac{x^2+2x-3}{4x} \div \frac{x^2+5x+6}{8x^2}$

23. $\frac{4}{x} + \frac{x}{3x+6} - \frac{3}{x^2+2x}$

24. $\frac{x+4}{x^2+2x+1} + \frac{x}{x^2-1} - \frac{2}{x-1}$

25. $\frac{4x-1}{x^2+x+1} - \frac{1}{2x-2} - \frac{x}{x^3-1}$

26. $\frac{\frac{6x}{x^2-4}}{\frac{3x-9}{2x+4}}$

27. $\frac{\frac{x^2-x-28}{3x^2-x-2}}{\frac{4x^2+16x+7}{3x^2+11x+6}}$

28. $\frac{\frac{1}{2}+\frac{2}{x-6}}{\frac{3x-6}{x^2-12x+36}}$

29. $\frac{\frac{x-2}{x+1}-\frac{x}{x-2}}{x+3}$

30. $\dfrac{\frac{x-3}{x^2+2x-8}}{\frac{2x+3}{x^2+9x+20}-\frac{1}{x-2}}$

31. $\dfrac{\frac{x-3}{x^3}-\frac{2}{x^3+x^2}}{\frac{1}{2}-\frac{1}{x^2}}$

32. $1-\dfrac{1}{1-\frac{1}{x}}$

33. $\dfrac{1-\frac{x}{x+1}}{2-\frac{x-1}{x}}$

Solve the equation.

34. $\dfrac{2x}{3x-5}-\dfrac{x+1}{3x+5}=\dfrac{-4}{9x^2-25}$

35. $\dfrac{x}{x^2-1}-\dfrac{x+3}{x^2-x}=\dfrac{-3}{x^2+x}$

36. $\dfrac{\frac{1}{x+1}}{\frac{1}{2}+\frac{x}{x+1}}=\dfrac{1}{5}$

37. $\dfrac{\frac{4}{x-2}+2}{\frac{6}{x}-\frac{3}{x-2}}=12$

38. $4x^3-8x=0$

39. $x^3+6x^2-7x=0$

40. $x^3+x^2-x-1=0$

41. $3x^3+4x^2=27x+36$

42. $x(2x-18)=-28$

43. $(2x+4)^2=144$

44. $3x^2+8x+1=0$

45. $2+\sqrt{12-2x}=x$

46. $\sqrt{5x+3}=-2$

State whether each equation is true or false.

47. $(p+q)^2=p^2+q^2$

48. $\sqrt{ab}=\sqrt{a}\sqrt{b}$

49. $\sqrt{a^2+b^2}=a+b$

50. $\frac{1+TC}{C}=1+T$

51. $\frac{1}{x-y}=\frac{1}{x}-\frac{1}{y}$

Determine the equation of the line described. Put your answer in the slope-intercept form, if possible.

52. Through $(-7,-9)$, perpendicular to $-4x-9y=-53$

53. Through $(-7,2)$, parallel to $-4x+6y=55$