Calculus BC Summer Work

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Find the smallest value in the domain of the function $f(x) = \sqrt{2x-5}$.
a. 2
b. $\frac{5}{2}$
c. 5
d. $\frac{2}{5}$
e. –2
f. 1
g. 0
h. –5

____ 2. Find the smallest value in the range of the function $f(x) = 3x^2 + 24x + 40$.
a. −4
b. −5
c. −6
d. −7
e. −8
f. −16
g. −24
h. −40

____ 3. Find the range of the function $f(x) = \begin{cases} x^2 - 4x & \text{if } x \le 2 \\ |x-4| & \text{if } x > 2 \end{cases}$.
a. [0,∞)
b. (–∞, 2]
c. [–4, ∞)
d. (–∞, 0]
e. [4, ∞)
f. (–∞, 4]
g. [2, ∞)
h. (–∞, –4]

____ 4. The function $f(x) = \sqrt{\frac{x-1}{x}}$ has as its domain all values of x such that
a. $x > 0$
b. $x \ge 1$
c. $x \le 0$
d. $x \le 1$
e. $0 < x \le 1$
f. $x \ge 1$ or $x < 0$
g. $x \ge -1$
h. $-1 \le x < 0$

____ 5. Let $f(x) = x^2 - 3x + 7$, then $f(2x)$ is equal to
a. $2x^2 - 6x + 7$
b. $4x^2 - 6x + 7$
c. $2x^2 - 6x + 14$
d. $4x^2 - 3x + 7$
e. $2x^2 + 6x - 7$
f. $4x^2 + 6x - 7$
g. $2x^2 - 3x + 7$
h. $4x^2 - 6x + 14$

____ 6. Let $h(x) = \sin^2 x + 3\sin x - 4$ and $g(x) = \sin x$. Find $f(x)$ so that $h(x) = (f \circ g)(x)$
a. $f(x) = (3x + 2)^2$
b. $f(x) = x + 3$
c. $f(x) = 3x^2 - 4$
d. $f(x) = x^2 - 3x + 4$
e. $f(x) = 3x^2 - 4x$
f. $f(x) = x^2 + 3x - 4$
g. $f(x) = x^2 - 4$
h. $f(x) = (x - 4)^2$

____ 7. Let $f(x) = 2 - x^3$ and $g(x) = 3 + x$. Find the value of $(f \circ g)(x)$ when $x = -5$.

a. −510
b. −5
c. −2
d. 0
e. 5
f. 10
g. 127
h. 130

____ 8. Relative to the graph of $y = x^2 + 2$, the graph of $y = 4x^2 + 2$ is changed in what way?

a. Compressed vertically by a factor of 1/2
b. Stretched horizontally by a factor of 2
c. Compressed horizontally by a factor of 1/2
d. Shifted 2 units upward
e. Shifted 2 units to the right
f. Stretched vertically by a factor of 2
g. Shifted 2 units to the left
h. Shifted 2 units downward

____ 9. For what value of x is $3^{4-x} = \sqrt{3}$?

a. 0
b. $\frac{1}{2}$
c. 1
d. $\frac{3}{2}$
e. 2
f. $\frac{5}{2}$
g. 3
h. $\frac{7}{2}$

____ 10. Find the inverse function for $f(x) = \frac{x-1}{x+1}$.

a. $\frac{x+1}{x-1}$
b. $\frac{x}{x+1}$
c. $\frac{x+1}{x}$
d. $\frac{1+x}{1-x}$
e. $\frac{x+1}{1-x}$
f. $\frac{x}{x-1}$
g. $\frac{x-1}{x+1}$
h. $\frac{x-1}{x}$

____ 11. Find the domain of the inverse for $f(x) = \sqrt{2x-5}$.

a. $(-\infty, -\frac{5}{2}]$
b. $(-\infty, 0]$
c. $\left[-\frac{5}{2}, \frac{5}{2}\right]$
d. $\left(-\infty, \frac{5}{2}\right]$
e. $\left[-\frac{5}{2}, \infty\right)$
f. $[0, \infty)$
g. $\left[\frac{2}{5}, \infty\right)$
h. $\left[\frac{5}{2}, \infty\right)$

____ 12. Find the range of the inverse for $f(x) = -\frac{3}{5+2x}$.

a. $\left(-\infty, -\frac{5}{2}\right)$
b. $(-\infty, 0)$
c. $\left(-\frac{5}{2}, \frac{5}{2}\right)$
d. $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2} \infty\right)$
e. $\left(-\frac{5}{2}, \infty\right)$
f. $(0, \infty)$
g. $\left(\frac{5}{2}, \infty\right)$
h. $\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2} \infty\right)$

____ 13. Given the function tan x with domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ find the domain of its inverse.

a. $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$
b. $[0, \infty)$
c. $[-\pi, \pi]$
d. $[-1, 1]$
e. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
f. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
g. $(-\infty, \infty)$
h. $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

____ 14. Find the value of $\log_{1/2} 1$.

a. -1
b. $-\frac{1}{2}$
c. 0
d. 10^2
e. 1
f. $\frac{1}{2}$
g. 2
h. -2

____ 15. Find the value of $\log_2 \frac{1}{8}$.

a. $\frac{1}{4}$
b. $\frac{1}{3}$
c. 0
d. 1
e. -1
f. 2
g. -2
h. -3

____ 16. Find the value of $\log_{16} 8$.

a. $\frac{1}{4}$
b. $\frac{1}{2}$
c. $\frac{3}{4}$
d. 1
e. $\frac{3}{2}$
f. 2
g. 3
h. 4

____ 17. Find the value of $\ln e^{e}$.

a. -1
b. $1/\sqrt{e}$
c. e
d. 0
e. $\sqrt{e}$
f. $1/e$
g. 1
h. $-e$

____ 18. Find the value of $\ln \sqrt{e^3}$.

a. $\frac{2}{3}$
b. $\sqrt{e}$
c. $e^3/2$
d. $\frac{3}{2}$
e. e^3
f. e^3-2
g. $2e/3$
h. $2/e^3$

____ 19. Find the value of $e^{3\ln 2}$.

a. $\frac{2}{3}$
b. $\frac{3}{2}$
c. 5
d. 6
e. 8
f. 9
g. 12
h. 18

____ 20. Solve the equation $\log_9(\ln x^3)=1$

a. $x=3^e$
b. $x=3e$
c. $x=e/3$
d. $x=1$
e. $x=e^2$
f. $x=1/e$
g. $x=3/e$
h. $x=e^3$

____ 21. Solve the equation $e^{2x-2}=4$.

a. $x=\ln 2$
b. $x=1-\ln 2$
c. $x=1+\ln 2$
d. $x=1-2\ln 2$
e. $x=1+2\ln 2$
f. $x=2+\ln 2$
g. $x=2-\ln 2$
h. $x=2-2\ln 2$

____ 22. Solve the equation $e^{2-3x}=125$.

a. $x=2-\ln 5$
b. $x=2-3\ln 5$
c. $x=\frac{2}{3}-\ln 5$
d. $x=\frac{2}{3}-3\ln 5$
e. $x=-\ln 5$
f. $x=-\frac{1}{3}\ln 5$
g. $x=\frac{2}{3}\ln 5$
h. $x=2+3\ln 5$

Short Answer

23. Let $f(x) = \sqrt{16 - x^2}$. Find

    (a) the domain of *f*.

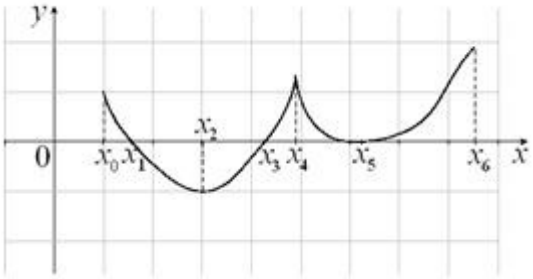
    (b) the range of *f*.

24. Let f (x) = $\begin{cases} x^2 + 3 & \text{if } x \le -1 \\ \dfrac{2 + 3x}{6} & \text{if } x > -1 \end{cases}$

    (a) the domain of *f*.

    (b) the range of *f*.

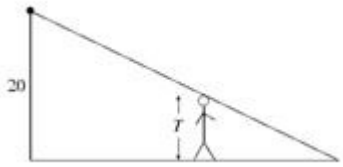
25. Given the graph of y = *f* (x):



    (a) *f* is increasing.

    (b) *f* is decreasing.

26. A parking lot light is mounted on top of a 20-foot tall lamppost. A person *T* feet tall is walking away from the lamppost along a straight path. Determine a function which expresses the length of the person's shadow in terms of the person's distance from the lamppost.



27. Let $f(x) = 8 + x^2$. Find each of the following:

(a) $f(2) + f(-2)$

(b) $f(x + 2)$

(c) $[f(x)]^2$

(d) $f(x^2)$

28. Let $f(x) = \sqrt{16 - x^2}$. Find each of the following:

(a) $f(0) + f(-2)$

(b) $f(x + 2)$

(c) $[f(x)]^2$

(d) $f(x^2)$

29. Let $f(x) = \sqrt{\frac{2}{x+3}}, x > -3$. Find each of the following:

(a) $f(-1) - f(-2)$

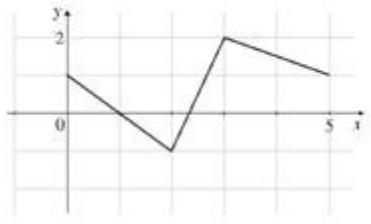
(b) $f(x^2 - 3)$

(c) $f(x^2) - 3$

(d) $[f(x - 3)]^2$

30. Evaluate the difference quotient $\frac{f(x) - f(a)}{x - a}$ for $f(x) = \frac{1}{x^2}$.

31. Given the graph of $y = f(x)$:



Sketch the graph of each of the following functions:

(a) $-f(x)$

(b) $f(-x)$

(c) $f(2x)$

(d) $2f(x)$

(e) $-f(-x)$

(f) $f\left(\frac{1}{2}x\right)$

(g) $\frac{1}{2}f(x)$

(h) $f(x+1)$

(i) $f(x)-1$

(j) $1-f(x)$

32. f and g are functions defined by the following table.

x	−3	−2	−1	0	1	2	3
$f(x)$	−5	−4	−3	−2	−1	−2	−3
$g(x)$	_4	1	−1	−2	−1	1	4

Determine the following:

(a) $(f + g)(2)$

(b) $(f - g)(-1)$

(c) $(f \cdot g)(0)$

(d) $(f/g)(3)$

(e) $(f \circ g)(-2)$

(f) $(f \circ f)(0)$

(g) $(g \circ f)(-1)$

(h) $(g \circ g)(-2)$

33. Find functions f and g such that $F(x) = 1 - 2\cos^2 x = (f \circ g)(x)$

34. Find functions f and g such that $F(x) = 1 \quad \sqrt{1 - \cos^2 x} = (f \circ g)(x)$

35. Find functions f and g such that $F(x) = e^{\sin x} = (f \circ g)(x)$

36. Sketch the graph of f for $f(x) = \sqrt[3]{x}$ and determine if f^{-1} exists . If so, find a formula for $f^{-1}(x)$ and sketch its graph.

37. Find the inverse of $f(x) = \dfrac{x+1}{2-x}$.

38. Find the value of $(4 \ln e^3 - e^{2\ln 3} + 5 \ln 1)(\ln \sqrt[4]{e})$.

39. Solve for x: $e^{2x-1} = e^{\ln 6}$.

40. Solve for $\ln x^2 = [\ln x]^2$.