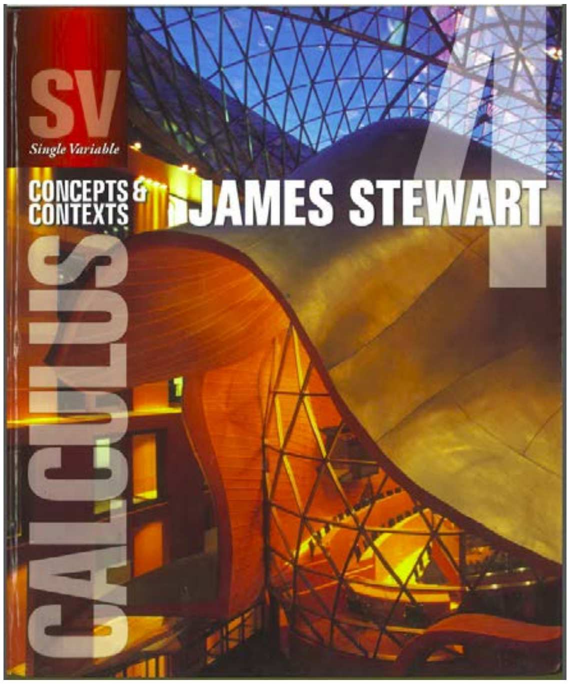
# AP Calculus AB Summer Math Packet (2019)




- This work will cover Chapter 1 in the book, which would be a review of the material for Pre-Calculus. The packet contains a brief review and example problems for some of the skills.
- It is due the 1st week of school. We will spend the first few days of school going over these problems. You are expected to have applied serious effort to all of them by day one.
- Please do not wait until the night before, day before, or weekend before school starts to do this!
- ALL WORK MUST BE SHOWN!
- There will be a quiz covering this material in the first week of school.
- Should you lose this packet, it can be found on the school web page.

Have a good summer. I look forward to seeing you in August.

Ms. Zareva

# Summary of Graphing Techniques

| To Graph: | Draw the Graph of $f$ and: | Functional Change to $f(x)$ |
|---|---|---|
| **Compressing or stretching** | | |
| $y = af(x)$, $a > 0$ | Multiply each $y$-coordinate of $y = f(x)$ by $a$.<br>Stretch the graph of $f$ vertically if $a > 1$.<br>Compress the graph of $f$ vertically if $0 < a < 1$. | Multiply $f(x)$ by $a$. |
| $y = f(ax)$, $a > 0$ | Multiply each $x$-coordinate of $y = f(x)$ by $\frac{1}{a}$.<br>Stretch the graph of $f$ horizontally if $0 < a < 1$.<br>Compress the graph of $f$ horizontally if $a > 1$. | Replace $x$ by $ax$. |



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| | | |
|---|---|---|
| **Reflection about the $x$-axis**<br>$y = -f(x)$ | Reflect the graph of $f$ about the $x$-axis. | Multiply $f(x)$ by $-1$. |
| **Reflection about the $y$-axis**<br>$y = f(-x)$ | Reflect the graph of $f$ about the $y$-axis. | Replace $x$ by $-x$. |

## Summary

### Properties of Logarithms

In the list that follows, $a > 0$, $a \neq 1$, and $b > 0$, $b \neq 1$; also, $M > 0$ and $N > 0$.

**Definition** $\quad y = \log_a x$ means $x = a^y$

**Properties of logarithms**

$\log_a 1 = 0; \quad \log_a a = 1$

$a^{\log_a M} = M; \quad \log_a a^r = r$

$\log_a(MN) = \log_a M + \log_a N$

$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

$\log_a M^r = r \log_a M$

If $M = N$, then $\log_a M = \log_a N$.

If $\log_a M = \log_a N$, then $M = N$.

**Change-of-Base Formula** $\quad \log_a M = \frac{\log_b M}{\log_b a}$

Given two functions $f$ and $g$, the **composite function**, denoted by $f \circ g$ (read as "$f$ composed with $g$"), is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers $x$ in the domain of $g$ such that $g(x)$ is in the domain of $f$.

## EXAMPLE 4 Finding the Exact Value of a Logarithmic Expression

Find the exact value of:

(a) $\log_2 16$ (b) $\log_3 \frac{1}{27}$

**Solution**

(a) To evaluate $\log_2 16$, think "2 raised to what power yields 16." So,

$$
\begin{aligned}
y &= \log_2 16 \\
2^y &= 16 && \text{Change to exponential form.} \\
2^y &= 2^4 && 16 = 2^4 \\
y &= 4 && \text{Equate exponents.}
\end{aligned}
$$

Therefore, $\log_2 16 = 4$.

(b) To evaluate $\log_3 \frac{1}{27}$, think "3 raised to what power yields $\frac{1}{27}$." So,

$$
\begin{aligned}
y &= \log_3 \frac{1}{27} \\
3^y &= \frac{1}{27} && \text{Change to exponential form.} \\
3^y &= 3^{-3} && \frac{1}{27} = \frac{1}{3^3} = 3^{-3} \\
y &= -3 && \text{Equate exponents.}
\end{aligned}
$$

Therefore, $\log_3 \frac{1}{27} = -3$.

## EXAMPLE 3 Writing a Logarithmic Expression as a Sum of Logarithms

Write $\log_a\left(x\sqrt{x^2+1}\right)$, $x > 0$, as a sum of logarithms. Express all powers as factors.

**Solution**

$$
\begin{aligned}
\log_a\left(x\sqrt{x^2+1}\right) &= \log_a x + \log_a \sqrt{x^2+1} && \log_a(M \cdot N) = \log_a M + \log_a N \\
&= \log_a x + \log_a (x^2+1)^{1/2} \\
&= \log_a x + \frac{1}{2}\log_a(x^2+1) && \log_a M^r = r \log_a M
\end{aligned}
$$

## EXAMPLE 1 Solving a Logarithmic Equation

Solve: $2 \log_5 x = \log_5 9$

**Solution**

The domain of the variable in this equation is $x > 0$. Because each logarithm is to the same base, 5, we can obtain an exact solution as follows:

$$
\begin{aligned}
2 \log_5 x &= \log_5 9 \\
\log_5 x^2 &= \log_5 9 && \log_a M^r = r \log_a M \\
x^2 &= 9 && \text{If } \log_a M = \log_a N, \text{ then } M = N. \\
x = 3 &\quad \text{or} \quad x = -3
\end{aligned}
$$

Recall that the domain of the variable is $x > 0$. Therefore, $-3$ is extraneous and we discard it.

## EXAMPLE 3 Solving a Logarithmic Equation

Solve: $\ln x = \ln(x + 6) - \ln(x - 4)$

**Solution** The domain of the variable requires that $x > 0$, $x + 6 > 0$, and $x - 4 > 0$. As a result, the domain of the variable here is $x > 4$. We begin the solution using the log of a difference property.

$$\ln x = \ln(x + 6) - \ln(x - 4)$$

$$\ln x = \ln\left(\frac{x + 6}{x - 4}\right) \qquad \ln M - \ln N = \ln\left(\frac{M}{N}\right)$$

$$x = \frac{x + 6}{x - 4} \qquad \text{If } \ln M = \ln N, \text{ then } M = N.$$

$$x(x - 4) = x + 6 \qquad \text{Multiply both sides by } x - 4$$

$$x^2 - 4x = x + 6 \qquad \text{Simplify.}$$

$$x^2 - 5x - 6 = 0 \qquad \text{Place the quadratic equation in standard form.}$$

$$(x - 6)(x + 1) = 0 \qquad \text{Factor.}$$

$$x = 6 \quad \text{or} \quad x = -1 \qquad \text{Zero-Product Property}$$

Since the domain of the variable is $x > 4$, we discard $-1$ as extraneous. The solution

## EXAMPLE 8 Solving an Exponential Equation

Solve: $e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$

**Solution** We use the Laws of Exponents first to get the base $e$ on the right side.

$$(e^x)^2 \cdot \frac{1}{e^3} = e^{2x} \cdot e^{-3} = e^{2x-3}$$

As a result,

$$e^{-x^2} = e^{2x-3}$$

$$-x^2 = 2x - 3 \qquad \text{Apply property (3).}$$

$$x^2 + 2x - 3 = 0 \qquad \text{Place the quadratic equation in standard form.}$$

$$(x + 3)(x - 1) = 0 \qquad \text{Factor.}$$

$$x = -3 \quad \text{or} \quad x = 1 \qquad \text{Use the Zero-Product Property.}$$

The solution set is $\{-3, 1\}$.

Solve: $5^{x-2} = 3^{3x+2}$

**Solution** Because the bases are different, we first apply property (6), Section 4.5 (take the natural logarithm of each side), and then use a property of logarithms. The result is an equation in $x$ that we can solve.

$$5^{x-2} = 3^{3x+2}$$

$$\ln 5^{x-2} = \ln 3^{3x+2} \qquad \text{If } M = N, \ln M = \ln N.$$

$$(x - 2)\ln 5 = (3x + 2)\ln 3 \qquad \ln M^r = r \ln M$$

$$(\ln 5)x - 2\ln 5 = (3\ln 3)x + 2\ln 3 \qquad \text{Distribute.}$$

$$(\ln 5)x - (3\ln 3)x = 2\ln 3 + 2\ln 5 \qquad \text{Place terms involving } x \text{ on the left.}$$

$$(\ln 5 - 3\ln 3)x = 2(\ln 3 + \ln 5) \qquad \text{Factor.}$$

$$x = \frac{2(\ln 3 + \ln 5)}{\ln 5 - 3\ln 3} \qquad \text{Exact solution}$$

$$\approx -3.212 \qquad \text{Approximate solution}$$

The solution set is $\left\{\frac{2(\ln 3 + \ln 5)}{\ln 5 - 3\ln 3}\right\}$.

1. Suppose $f(x) = x^2 - 3x + 4$ and $g(x) = x - e^x$. Find

a) $(f + g)(3)$

b) $(fg)(5)$

c) $f(2x)$

d) $f(3x + 1)$

e) $(f \ o \ g)(4)$

f) $(g \ o \ f)(4)$

g) Find $f(u)$, where $u = \sin x$

2. Sketch the graph of $y = 2e^{x+3} - 4$

3. Simplify:

a) $\ln\sqrt{e^3}$

b) $e^{3\ln 2}$

c) $\ln(64e^3) - \ln(4e)$

4. Solve the following equations

a) $e^{2x-2} = 4$

b) $\ln(x^2) = (\ln x)^2$

5. Simplify the following expressions

a) $\dfrac{1+\frac{1}{x}}{x-1}$

b) $\dfrac{x+1}{\frac{1}{x}-1}$

6. a) find an equation for the line through the points $(2,5)$ and $(-3,4)$

b) find an equation for the line through $(2,-5)$ with slope 6

c) find an equation for the line through $(4,1)$ parallel to the line $3x + 2y = 6$

d) find an equation for the line through $(-3,2)$ perpendicular to the line $y = -2x + 6$

7. For each of the following functions $f$, compute and simplify the expression $\frac{f(x+h)-f(x)}{h}$

a) $f(x) = 3x - 5$

b) $f(x) = x^2 - 3x + 1$

c) $f(x) = \sqrt{x}$

d) $f(x) = \frac{1}{x+5}$

8. Find the exact value of each expression.

a) $\sin\left(\frac{10\pi}{3}\right)$

d) $\log_{16} 8$

b) $\sin\frac{\pi}{2} - \tan\frac{11\pi}{4}$

e) $\ln e^e$

c) $\cot(3\pi)$

f) $\log_2 e - \log_2(e/16)$

9.

(a) Solve $3^{4-x} = \sqrt{3}$

(b) For what numbers $x$, on the interval $-2\pi \leq x \leq 2\pi$ , does $\sin x = 1$ ?

(c) Solve $(2x+4)^2 = 144$

(d) Solve $x^3 + x^2 - x - 1 = 0$

10.

a) Graph $f(x) = 2x^2 - 3x + 2$

b) Determine where $f$ is increasing and decreasing

11. Determine whether the quadratic function $f(x) = -x^2 + 4x + 5$ has a maximum or minimum value. Then find the maximum or minimum value.

12. Graph the following piece-wise defined functions:

a) $f(x) = |x - 3|$

b) $f(x) = \begin{cases} 0 & if\ x < 0 \\ 1 & if\ x \geq 0 \end{cases}$

c) $g(x) = \begin{cases} x + 1 & if\ x \neq 1 \\ \pi & if\ x = 1 \end{cases}$

d) $f(x) = \begin{cases} \frac{1}{x^2} & if\ x \neq 0 \\ 1 & if\ x = 0 \end{cases}$

e) $f(x) = \begin{cases} x^2 & if\ x < 1 \\ \ln x + 1 & if\ 1 \leq x < e \\ x & if\ x \geq e \end{cases}$